Scalar-Tensor Gravity Theory For Dynamical Light Velocity

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Abstract

A gravity theory is developed with the metric $\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$. In the present universe the additional contribution from the scalar field in the metric $\hat{g}_{\mu\nu}$ can generate an acceleration in the expansion of the universe, without negative pressure and with a zero cosmological constant. In this theory, gravitational waves will propagate at a different speed from non-gravitational waves. It is suggested that gravitational wave experiments could test this observational signature.

1 Introduction

Recent observations of apparent luminosities of Type Ia supernovae (SNe-Ia) at moderate redshift ($z \sim 0.6$) indicate that the universe is expanding at an accelerated rate [1]. If this observational evidence is correct, then the implications for cosmology are remarkable. Attempts to explain this phenomenon include "quintesssence" [2], the cosmological constant [3], a domain wall dominated universe [4] and non-perturbative vacuum contributions to the effective action of a massive scalar field [5]. In general it would appear

that the cosmological fluid is dominated in the present universe by an exotic energy density, which has a negative pressure and which did not play an important role in the early universe. The fine tuning problem related to the cosmological constant is well-known [3, 6] and amounts to a fine tuning of order 10^{100} between the early universe inflationary phase and the present universe. For the quintessence model described by a slowly rolling scalar field, the potential has to be extremely flat, so that it cannot roll to its true minimum in the present universe. Characterizing the flatness by a mass m requires that the mass be extraordinarily ultra-light $m \sim 10^{-33}$ eV. A mass of the same size is implied by the non-perturbative vacuum driven mechanism.

All standard gravity theories, including Einstein's general relativity (GR), are normally required to satisfy the positive energy conditions for matter in the present universe, since over extended times this is perceived as a reasonable physical requirement of a gravity theory. For the vacuum energy, $p = -\rho$, which leads to a cosmological constant Λ. It is believed that symmetry principles exist in particle physics which would force Λ to be zero, but there have been no convincing arguments found to support this. How can we then describe the apparent speeding up of the expansion of the universe in a gravity theory without violating the positivity conditions on the density and pressure? Can we construct a self-consistent gravity theory which can explain the data without violating the positivity conditions and with a zero cosmological constant? In the following, we shall develop such a gravity theory based on a metric given by

$$\hat{g}_{\mu\nu} = A[\phi]g_{\mu\nu} + B[\phi]\partial_\mu\phi\partial_\nu\phi, \tag{1}$$

where ϕ is a scalar field and $\partial_\mu\phi = \partial\phi/\partial x^\mu$. The inverse metrics $\hat{g}^{\mu\nu}$ and $g^{\mu\nu}$ satisfy

$$\hat{g}^{\mu\alpha}\hat{g}_{\nu\alpha} = \delta^\mu{}_\nu, \quad g^{\mu\alpha}g_{\nu\alpha} = \delta^\mu{}_\nu. \tag{2}$$

When $A[\phi] = 1$ and $B[\phi] = 0$, we retrieve conventional GR. As an immediate simplification, we assume that $A[\phi] = 1$ and $B[\phi] = B =$ constant, which serves to eliminate contributions to the field equations for ϕ with more complicated dependence on derivatives of the source tensor. The assumption that $A[\phi] = 1$ is motivated by the fact that the, essentially, conformal factor has been exhaustively studied in the Brans-Dicke scenario [7, 8] and we are interested in novel effects.

In a previous work [9], a bimetric gravity theory was constructed based on a metric similar to (1) but in which the second term was described by a vector

field ψ_μ. This model provided a dynamical mechanism for a superluminary theory in which light travels faster in the early universe, thereby resolving problems in cosmology [9, 10, 11, 12].

In the gravity theory presented in the following, we shall find that there is an extra contribution to the gravity component in the equations of motion for the expansion factor in cosmology. This contribution can lead to an acceleration of the expansion of the universe. The equation of state satisfies the standard positivity conditions for the density and pressure. A fit to the type Ia supernovae (SNe-Ia) data is obtained.

2 The Action and Field Equations

The model that we consider here is identical in spirit to that which appeared in an earlier publication [9], with the important difference that the coupling is through a scalar field which, given the predominance of scalar fields in cosmological models [13] and as effective models of more fundamental theories such as string theory, is more in line with current research in the field. Here we will constrain ourselves to the simplest of the class of such models so that we focus on novel effects and avoid confusing the issue with Brans-Dicke-like and dilaton couplings. These are easily included.

The model consists of three parts, represented by the three separate contributions to the action:

$$S = S_{\mathrm{grav}} + S_\phi + S_{\mathrm{M}}. \tag{3}$$

The standard general relativity contribution is:

$$S_{\mathrm{grav}} = -\frac{1}{\kappa}\int d^4x\sqrt{-g}(R[g] - 2\Lambda), \tag{4}$$

where $\kappa = 16\pi G/c^4$, Λ is the cosmological constant, and we employ a metric with a $(+---)$ signature. We also have a contribution from a minimally-coupled scalar field:

$$S_\phi = -\frac{1}{\kappa}\int d^4x\sqrt{-g}\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\Big], \tag{5}$$

where the scalar field ϕ has been chosen to be dimensionless. The stress-energy tensor of the scalar field is of the standard form:

$$T_\phi^{\mu\nu} = \frac{1}{\kappa}\Big[g^{\mu\alpha}g^{\nu\beta}\partial_\alpha\phi\partial_\beta\phi - g^{\mu\nu}\Big(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\Big)\Big], \tag{6}$$

and results from the variation of the scalar field action with respect to $g_{\mu\nu}$.

Where our model departs from conventional model-building wisdom is in the coupling of the gravitational field to matter. Instead of constructing the matter action using the metric $g_{\mu\nu}$, we use the combination (1), resulting in the identification of $\hat{g}_{\mu\nu}$ as the physical metric that provides the arena on which matter fields interact. The matter action, $S_{\rm M}[\psi^I] = S_{\rm M}[\hat{g}, \psi^I]$, where ψ^I represents all the matter fields in spacetime, is one of the standard forms, and therefore the energy-momentum tensor derived from it is given by

$$\frac{\delta S_{\rm M}}{\delta \hat{g}_{\mu\nu}} = -\frac{1}{2}\sqrt{-\hat{g}}\hat{T}^{\mu\nu}. \tag{7}$$

It satisfies the conservation laws

$$\hat{\nabla}_\nu\Big[\sqrt{-\hat{g}}\hat{T}^{\mu\nu}\Big] = 0, \tag{8}$$

as a consequence of the matter field equations only. Note that it is the covariant derivative $\hat{\nabla}_\mu$ which is compatible with the metric $\hat{g}_{\mu\nu}$ which appears, and *not* ∇_μ which is defined to be compatible with $g_{\mu\nu}$. We have included the factor $\sqrt{-\hat{g}}$ in (8) since this will be a convenient starting point to derive the consistency of the Bianchi identities with the field equations.

As an explicit example, if the matter model consisted of a Maxwell one-form field, then we would have:

$$S_M = -\int d^4x\sqrt{-\hat{g}}\Big[\frac{1}{4}\hat{g}^{\mu\nu}\hat{g}^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}\Big], \tag{9}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Note that we have assumed that it is the density $\sqrt{-\hat{g}}$ that appears in the action, which implies that the energy-momentum tensor

$$\hat{T}^{\alpha\beta} = \hat{F}^{\alpha\mu}\hat{F}^{\beta}{}_{\mu} - \frac{1}{4}\hat{F}^2\hat{g}^{\alpha\beta}, \tag{10}$$

where for example $\hat{F}^{\alpha\beta} = \hat{g}^{\alpha\mu}\hat{g}^{\beta\nu}F_{\mu\nu}$, satisfies (8) if A_μ satisfies the field equations $\hat{\nabla}_\beta\hat{F}^{\alpha\beta} = 0$.

Since it is only $\hat{g}_{\mu\nu}$ which is 'visible' to matter fields, it is a reasonable assumption (and in line with the identical assumption implicitly made in general relativity) that material test bodies will follow geodesics of $\hat{g}_{\mu\nu}$:

$$\frac{d\hat{u}^\alpha}{d\lambda} + \hat{\Gamma}^\alpha_{\mu\nu}\hat{u}^\mu\hat{u}^\nu = 0, \tag{11}$$

where λ is an affine parameter and the tangent vector $\hat{u}^\mu$ is normalized as: $\hat{g}_{\mu\nu}\hat{u}^\mu\hat{u}^\nu = 0$, $\hat{g}_{\mu\nu}\hat{u}^\mu\hat{u}^\nu = c^2$ for null and time-like geodesics, respectively.

Because all matter fields will couple to $\hat{g}_{\mu\nu}$ in the same manner, the Weak Equivalence Principle is not violated. We could easily introduce weak equivalence principle violating terms into the theory through Yukawa couplings between the scalar field ϕ and the matter fields. This we leave for future consideration. Because one can always work in a locally defined frame with $\hat{g}_{\mu\nu} \approx \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski flat spacetime metric tensor, the field equations for the matter fields can take on their special relativistic form and the Einstein Equivalence Principle is not violated, either. However, if one considers expanding the matter and gravitational fields in some region of spacetime where $\hat{g}_{\mu\nu} \approx \eta_{\mu\nu}$, then the perturbation equations for $\hat{g}_{\mu\nu}$ and ϕ will in general *not* take on their special relativistic form, and therefore the Strong Equivalence Principle is expected to be violated.

One key feature of our model is that there are no instabilities induced by this coupling, in the sense of higher-order derivatives or couplings to unhealthy gauge modes. To see this we derive the field equations.

From the variation of the metric (1) we get

$$\delta\hat{g}_{\mu\nu} = \delta g_{\mu\nu} + 2B\partial_{(\mu}\phi\partial_{\nu)}\delta\phi, \tag{12}$$

and using the definition (7), we obtain the field equations

$$G^{\mu\nu} = \frac{\kappa}{2}(T^{\mu\nu}_\phi + s\hat{T}^{\mu\nu}), \tag{13}$$

$$\nabla^2\phi + V'[\phi] - \kappa s B\hat{T}^{\mu\nu}\hat{\nabla}_\mu\hat{\nabla}_\nu\phi = 0, \tag{14}$$

where $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$, $s = \sqrt{-\hat{g}}/\sqrt{-g}$ and $\nabla^2\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$.

As a check that we have indeed produced sensible field equations, we can show that the Bianchi identities on the curvature of $g_{\mu\nu}$ are compatible with the field equations. From the definition (1) we can determine the inverses

$$\hat{g}^{\mu\nu} = g^{\mu\nu} - \frac{B}{I}\nabla^\mu\phi\nabla^\nu\phi, \tag{15}$$

and

$$g^{\mu\nu} = \hat{g}^{\mu\nu} + \frac{B}{K}\hat{\nabla}^\mu\phi\hat{\nabla}^\nu\phi, \tag{16}$$

where

$$I = 1 + Bg^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \quad K = 1 - B\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \tag{17}$$

from which it follows that $IK = 1$, and we have defined $\nabla^\mu \phi = g^{\mu\nu}\partial_\nu \phi$ and noted that

$$\hat{\nabla}^\mu \phi = \hat{g}^{\mu\nu}\partial_\nu \phi = K\nabla^\mu \phi. \tag{18}$$

Using these in the definition of the metric compatible connection coefficients $\hat{\Gamma}^\alpha_{\mu\nu}$, we find the equivalent forms

$$\hat{\Gamma}^\alpha_{\mu\nu} - \Gamma^\alpha_{\mu\nu} = \frac{B}{I}\nabla^\alpha\phi\nabla_\mu\nabla_\nu\phi = \frac{B}{K}\hat{\nabla}^\alpha\phi\hat{\nabla}_\mu\hat{\nabla}_\nu\phi. \tag{19}$$

The matter energy-momentum conservation laws (8) can be re-written as

$$\hat{\nabla}_\nu\Big[\sqrt{-\hat{g}}\hat{T}^{\mu\nu}\Big] = \nabla_\nu\Big[\sqrt{-\hat{g}}\hat{T}^{\mu\nu}\Big] + (\hat{\Gamma}^\mu_{\alpha\beta} - \Gamma^\mu_{\alpha\beta})\sqrt{-\hat{g}}\hat{T}^{\alpha\beta} = 0, \tag{20}$$

which using (19), become

$$\hat{\nabla}_\nu\Big[\sqrt{-\hat{g}}\hat{T}^{\mu\nu}\Big] = \nabla_\nu\Big[\sqrt{-\hat{g}}\hat{T}^{\mu\nu}\Big] + \frac{B}{K}\hat{\nabla}^\mu\phi\sqrt{-\hat{g}}\hat{T}^{\alpha\beta}\hat{\nabla}_\alpha\hat{\nabla}_\beta\phi = 0. \tag{21}$$

From the ∇_ν-derivative of (6) we find

$$\nabla_\nu[\sqrt{-g}T^{\mu\nu}_\phi] = \frac{1}{\kappa}\sqrt{-g}(\nabla^2\phi + V'[\phi])\nabla^\mu\phi = \sqrt{-\hat{g}}\frac{B}{K}\hat{\nabla}^\mu\phi\hat{T}^{\alpha\beta}\hat{\nabla}_\alpha\hat{\nabla}_\beta\phi, \tag{22}$$

where we have used (18) and the scalar field equations (14). Finally, taking the ∇_ν-derivative of ($\sqrt{-g}$ times) the field equations (13) we find

$$\nabla_\nu[\sqrt{-g}G^{\mu\nu}] = \frac{\kappa}{2}\Big(\nabla_\nu[\sqrt{-g}T^{\mu\nu}_\phi] + \nabla_\nu[s\sqrt{-g}\hat{T}^{\mu\nu}]\Big), \tag{23}$$

the right hand side of which vanishes using (21) and (22). We have therefore shown that (8) is sufficient to guarantee that the field equations (13) are consistent with the Bianchi identities.

Although we have introduced two metrics into the action, the field equation for ϕ (equation (14)) is forcing upon us another null cone. In order to identify it, we use (19) to find

$$\hat{\nabla}_\mu\hat{\nabla}_\nu\phi = \frac{1}{I}\nabla_\mu\nabla_\nu\phi, \tag{24}$$

which gives the equivalent form of (14):

$$\Big(g^{\mu\nu} - s\kappa\frac{B}{I}\hat{T}^{\mu\nu}\Big)\nabla_\mu\phi\nabla_\nu\phi + V'[\phi] = 0. \tag{25}$$

Therefore, there are three light cones in our gravitational theory. The light cone, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0$, which describes the null surfaces of the gravitational propagation, $d\hat{s}^2 = 0$ which describes the propagation of ordinary matter, and $d\bar{s}^2 = 0$ determined from the metric

$$\bar{g}^{\mu\nu} = g^{\mu\nu} - s\kappa\frac{B}{I}\hat{T}^{\mu\nu}, \tag{26}$$

which describes the propagation of the scalar field ϕ obtained from the principal part of (25). There are, of course, expressions for these in terms of the metric $\hat{g}_{\mu\nu}$, but they will not be required in their general form here.

We stress that the field equations do not "suffer" from any higher-order derivative instabilities, and the causality 'violation' is expressed in this class of models as multiple light cones dynamically determined indirectly by the matter fields. Thus it can be said that it is a reasonable model with a "variable speed of light".

3 Cosmological Model

We shall assume that spacetime and the scalar field ϕ are homogeneous and isotropic. We will also begin by writing the metric $g_{\mu\nu}$ in comoving form, which leads us to the standard Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = c^2dt^2 - R^2(t)\Big[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\Big], \tag{27}$$

where we employ a dimensionless radial coordinate r and $k = 0, \pm 1$ for the flat, closed and hyperbolic spatial topologies, respectively. Let us write the metric as

$$ds^2 = c^2dt^2 - R^2(t)\gamma_{ij}dx^i dx^j. \tag{28}$$

For the metric $\hat{g}_{\mu\nu}$ we obtain

$$d\hat{s}^2 = I(t)c^2dt^2 - R^2(t)\gamma_{ij}dx^i dx^j, \tag{29}$$

where

$$I(t) = 1 + \frac{B}{c^2}\dot{\phi}^2(t), \tag{30}$$

and $\dot{\phi} = d\phi/dt$. We have that $s = \sqrt{I}$ and that $I > 0$ to guarantee that the metric $\hat{g}_{\mu\nu}$ is real and non-degenerate. From (26), we find

$$d\bar{s}^2 = \Big(1 - \frac{c^2\kappa B}{I^{3/2}}\rho_M\Big)c^2dt^2 - \Big(1 + \frac{\kappa B}{I^{1/2}}p_M\Big)R^2(t)\gamma_{ij}dx^i dx^j, \tag{31}$$

and so we have that $I^{3/2} > c^2\kappa B\rho_M$ for this metric to be non-degenerate.

Let us assume a perfect fluid model for the energy-momentum tensor

$$\hat{T}^{\mu\nu} = (\rho_M + \frac{p_M}{c^2})\hat{u}^\mu\hat{u}^\nu - p_M\hat{g}^{\mu\nu}, \tag{32}$$

where $\hat{u}^\mu = dx^\mu/d\hat{s}$ is normalized to $\hat{g}_{\mu\nu}\hat{u}^\mu\hat{u}^\nu = c^2$, so that the only non-vanishing component is $\hat{u}^0 = c/\sqrt{\hat{g}_{00}} = c\sqrt{\hat{g}^{00}}$. We obtain

$$\hat{T}^{00} = \frac{\rho_M}{I}, \quad \hat{T}^{0i} = 0, \quad \hat{T}^{ij} = \frac{p_M}{R^2}\gamma^{ij}, \tag{33}$$

and from (8) the matter conservation equation:

$$\dot{\rho}_M + 3\Big(\rho_M + \frac{p_M}{c^2}\Big)\frac{\dot{R}}{R} = 0. \tag{34}$$

The field equations (13) become

$$\Big(\frac{\dot{R}}{R}\Big)^2 + \frac{c^2k}{R^2} = \frac{1}{3}c^2\Lambda + \frac{1}{6}\Big(\frac{1}{2}\dot{\phi}^2 + c^2V[\phi]\Big) + \frac{\kappa c^4}{6}\frac{\rho_M}{\sqrt{I}}, \tag{35}$$

$$\Big(\frac{\dot{R}}{R}\Big)^2 + 2\frac{\ddot{R}}{R} + \frac{c^2k}{R^2} = c^2\Lambda - \frac{1}{2}\Big(\frac{1}{2}\dot{\phi}^2 - c^2V[\phi]\Big) - \frac{\kappa c^2}{2}\sqrt{I}p_M, \tag{36}$$

and the scalar field equation (14) reduces to

$$\frac{1}{c^2}\Big(1 - \frac{c^2\kappa B}{I^{3/2}}\rho_M\Big)\ddot{\phi} + \frac{3\dot{R}}{c^2R}\dot{\phi}\Big(1 + \frac{\kappa B}{\sqrt{I}}p_M\Big) + V'[\phi] = 0. \tag{37}$$

The field equations (35-37) are written in a comoving frame for the gravitational metric $g_{\mu\nu}$. In order to make a more obvious connection with standard cosmology, we shall transform our field equations by using the time coordinate defined by

$$\tau = \int \sqrt{I}dt, \tag{38}$$

which puts the matter metric $\hat{g}_{\mu\nu}$ into comoving coordinate form. In this new frame, the field equations become

$$\Big(\frac{\dot{R}}{R}\Big)^2 + \frac{c^2kK}{R^2} = \frac{1}{3}c^2\Lambda K + \frac{1}{6}\Big(\frac{1}{2}\dot{\phi}^2 + c^2KV[\phi]\Big) + \frac{1}{6}\kappa c^4K^{3/2}\rho_M, \tag{39}$$

$$\left(\frac{\dot{R}}{R}\right)^2+2\frac{\ddot{R}}{R}+\frac{c^2kK}{R^2}=\frac{\dot{K}\dot{R}}{KR}+c^2\Lambda K-\frac{1}{2}\Big(\frac{1}{2}\dot{\phi}^2-c^2KV[\phi]\Big)-\frac{1}{2}\kappa c^2\sqrt{K}p_M, \quad (40)$$

and

$$\frac{1}{c^2}\Big(1-\kappa c^2BK^{3/2}\rho_M\Big)\ddot{\phi}+\frac{3K\dot{R}}{c^2R}\dot{\phi}\Big(1+\kappa B\sqrt{K}p_M\Big)+K^2V'[\phi]=0. \quad (41)$$

Making the definitions

$$\rho_\phi=\frac{1}{\kappa c^2}\Big(\frac{1}{2c^2}\dot{\phi}^2+KV[\phi]\Big), \quad p_\phi=\frac{1}{\kappa}\Big(\frac{1}{2c^2}\dot{\phi}^2-KV[\phi]\Big), \quad (42)$$

we see that $K=1-\kappa c^2B(\rho_\phi+p_\phi/c^2)$, and that $\rho_B:=1/(\kappa c^2B)$ gives the energy scale at which K deviates significantly from 1, and therefore significant deviations from a standard general relativity plus matter model are to be expected.

Defining: $\rho_\Lambda=2\Lambda/(c^2\kappa)$, $p_\Lambda=-2\Lambda/\kappa$, $\Omega_k=kc^2/(R^2H^2)$, $\Omega_\Lambda=c^4\kappa\rho_\Lambda/(6H^2)$, $\Omega_\phi=c^4\kappa\rho_\phi/(6H^2)$ and $\Omega_M=c^4\kappa\rho_M/(6H^2)$, we can write the Friedmann equation (39) in the "sum-rule" form:

$$1+K\Omega_k=K\Omega_\Lambda+\Omega_\phi+K^{3/2}\Omega_M. \quad (43)$$

Although this form is similar to that which appears in standard cosmological models, the factor K and Ω_ϕ depend on the scalar field and therefore this is *not* just a simple sum of individual energy contributions. From (40) we can derive an expression for the deceleration parameter:

$$q=-\frac{\ddot{R}}{H^2R}=-\frac{\dot{K}}{2HK}+\frac{1}{2}(1+K\Omega_k)+\frac{c^2\kappa}{4H^2}(p_\phi+K^{1/2}p_M+Kp_\Lambda). \quad (44)$$

The first term can be evaluated by using the scalar field equation (41) to give

$$\frac{\dot{K}}{2HK}=\frac{3(1-K)(1+\kappa BK^{1/2}p_M)+H^{-1}B\dot{\phi}KV'[\phi]}{1-\kappa c^2BK^{3/2}\rho_M}. \quad (45)$$

Note that the denominator is positive-definite wherever the metric (31) has the correct signature, the first term in the numerator is positive-definite whenever the metric (27) has the correct signature, and the second term in the numerator is positive provided that the universe is expanding $H>0$ and $\partial_tV[\phi]>0$.

4 Parameterization of the Cosmological Model

We shall now consider the present universe modeled by a spatially flat FRW spacetime containing non-relativistic matter and the scalar field ϕ. We shall assume the standard equation of state with negligible matter pressure $p_M \approx 0$, so that from the conservation equation (34) we get

$$\frac{\rho_M}{\rho_{0\,M}} = \left(\frac{R_0}{R}\right)^3, \tag{46}$$

where $\rho_{0\,M} = \rho_M(t_0)$, $R_0 = R(t_0)$ denote the present values of the density and cosmic scale factor with t_0 denoting the present value of time t. Taking $k = \Lambda = 0$, Eq. (43) becomes

$$1 = \Omega_{0\,\phi} + K_0^{3/2}\Omega_{0\,M}. \tag{47}$$

We can obtain a fit to the present data by essentially following the method of Perlmutter et al. [1] and choosing for a spatially flat universe the best fit value: $\Omega_{0\,M} = 0.28$ and the parameterization:

$$\Omega_{0\,\phi} = 1 - 0.28K_0^{3/2}. \tag{48}$$

The present value of the deceleration parameter can be written

$$q_0 = -\frac{\dot{K}_0}{2H_0K_0} + \frac{1}{2} + \frac{c^2\kappa}{4H_0^2}p_{0\,\phi}. \tag{49}$$

We see that in our gravity theory, we can achieve a negative deceleration parameter q_0, if the first term on the right-hand side dominates. We stress that the cosmic acceleration with $\ddot{R}(t) > 0$ is *caused by the dynamics of the scalar field* ϕ, which acts as a gravitational field component, and the matter pressure p_M can be negligible and the cosmological constant Λ can be zero. Galaxy formation at earlier times can be obtained for small and positive q, if the solution for ϕ and the pressure p_ϕ are appropriately chosen. As has been demonstrated elsewhere, the standard horizon and flatness problems can also be resolved by superluminary models [9, 10, 11, 12]. In the present theory, an early universe inflationary expansion phase can be obtained by a suitable choice of initial conditions for ϕ and $V(\phi)$. A solution that accomplishes this has been found and will be presented in a forthcoming article.

5 Experimental Signature for Gravitational Waves

Let us define

$$\bar{c} = c\sqrt{K} = c(1 - \frac{B}{c^2}\dot{\phi}^2)^{1/2}, \quad \bar{G} = GK^{3/2} = G(1 - \frac{B}{c^2}\dot{\phi}^2)^{3/2}, \tag{50}$$

and

$$\bar{\Lambda} = \Lambda + \frac{1}{2}\Big(\frac{1}{2\bar{c}^2}\dot{\phi}^2 + V[\phi]\Big), \tag{51}$$

so that we can rewrite (39) as

$$H^2 + \frac{\bar{c}^2 k}{R^2} = \frac{1}{3}\bar{c}^2\bar{\Lambda} + \frac{8\pi\bar{G}}{3}\rho_M. \tag{52}$$

This has the form of the Friedmann equation in Einstein gravity with an "effective" velocity of light $\bar{c}$, gravitational constant $\bar{G}$ and cosmological constant $\bar{\Lambda}$. We have therefore mapped our model to a particular case of the models considered in [11]. It is interesting to note that in this particular model $\bar{G}/\bar{c}^3 = G/c^3 = \text{constant}$.

These effective time varying constants are merely definitions that allow us to write the Friedman equation in the standard form. Although $\bar{G}$ may be interpreted as an effective gravitational coupling to matter, we must be careful how we interpret $\bar{c}$ as a time varying speed of light. In fact, in this frame matter field perturbations will propagate at a speed determined by the line element $d\hat{s}^2 = c^2dt^2 - R^2(t)\gamma_{ij}dx^idx^i$, whereas metric perturbations will propagate at a speed determined by $ds^2 = \bar{c}^2dt^2 - R^2(t)\gamma_{ij}dx^idx^i$; in both cases $R(t)$ is the solution of (52). This means that matter fields will behave as in conventional models, but the gravitational field cannot be understood in terms of a minimal coupling to these fields. Note that this rules out effects like those considered in [15].

We can now predict that there will be a time lag Δt between the speed of gravitational waves travelling along the null surface of the gravitational light cone $ds^2 = 0$ and photons and other matter particles travelling through and on the null surface of the matter light cone $d\hat{s}^2 = 0$. This could be the basis of an important observational signature for our gravity theory, for with $K \approx 0$ in the early universe the time lag Δt could be a measurable quantity in gravitational wave experiments. Morover, the effective gravitational constant $\bar{G}$ would by the same reasoning be smaller in the early universe than its presently measured value G.

6 Conclusions

We have developed a gravity theory based on a bimetric structure. When $B = 0$, we regain standard GR. There are three light cones associated with the metrics $g_{\mu\nu}$, $\hat{g}_{\mu\nu}$ and $\bar{g}_{\mu\nu}$. All matter fields except the scalar field ϕ propagate in the geometry described by $\hat{g}_{\mu\nu}$ and material test particles and photons will propagate along geodesics determined by this geometry and obey the equivalence principle. In future work the properties and predictions of static spherically symmetric solutions for gravitational phenomena will be investigated. The theory could have significance for the problem of singularities and the physical properties of black holes. There is an interesting experimental signature in our gravity theory, due to the slowing down of gravitational waves emitted in the early universe which could be a detectable physical phenomenon in gravitational wave experiments.

The cosmological model is in agreement with the magnitude-redshift relation of Type Ia supernovae for $\Omega_{0\,M} = 0.28$. In a recent paper, Caldwell [14] considered the possibility that future observational constraints and more accurate supernovae data may require that quintessence models may need significantly more negative values for the pressure in the equation of state. In our model this would correspond to an increase in the size of $\Omega_{0\,\phi}$ keeping the density positive and $p_M \approx 0$.

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